

How was Mathematics modernized?

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Modernization of Mathematics

- The previous talk is about modernism in mathematics.
- This talk is about the modernization of mathematics.
- They are *different* although they are closely related.
 - My research on modernization of math. profited much by the modernism idea by Prof. Gray as they are related.
- The modernization of mathematics, as I call it, is the transformation of the attitudes of mathematicians which was completed after the second world war.

What is Modernization of Mathematics? 1/3

- The modernization theory was originated by the German sociologist Max Weber, and influenced many sociologists such as T. Parsons and A. Giddens, etc..
- My modernization theory of mathematics is also based on Weber-sociology, and can be seen as a particular case of Weber's modernization theory of societies.
- Weber's modernization idea has many elements such as secularization, formal rationality, warring deities, etc..
- Each specific domain of society has its own characteristic of modernization, as Weber defined the concept of formal rationality in different ways for justice and economy.

What is Modernization of Mathematics? 2/3

- The main characteristics of the modernization of mathematics are secularization and formal rationality.
- Since I do not have enough time to explain the whole picture, in this talk, I will concentrate on the first issue “secularization of mathematics”.
- Secularization of mathematics?
- I am not talking about religious values in mathematics.
- Secularization of mathematics in my sense means that mathematicians distance themselves more and more from philosophy.

What is Modernization of Mathematics? 3/3

- Thus modernized mathematicians would say:
No Philosophy Please, We're Mathematicians.
- We will see a typical example of the modernization of mathematics in an episode between a secularized great mathematician and a non-secularized great mathematician, and will see the meaning of the sentence “No Philosophy Please, We're Mathematicians” in the episode.
- The non-secularized great mathematician is Hermann Weyl, on whom the previous talk was.

Weyl and Kodaira

- Weyl, on the right, was the hero for Kunihiro Kodaira, on the left, Japan's first Fields medalist, who was awarded the Fields medal for his generalization of Weyl's theory of Riemann surfaces.



K. Kodaira
小平邦彦
1915-1997



H. Weyl
1885-1955

Kodaira's "Memories of Princeton"

- Kodaira wrote in his essay "Memories of Princeton (プリンストンの思い出)", 1981 August issue, Sugaku Seminar (数学セミナー), a popular magazine of mathematics as follows:
- As a student, Kodaira was fascinated by Weyl's famous booklet "[Die Idee der Riemannschen Fläche](#)" ([The Concept of a Riemann Surface](#)) 1913.
- Kodaira extend Weyl's theory of one-dimensional Riemann surfaces of the booklet to n -dimensional surfaces.
- Weyl appreciated this work, and invited Kodaira to the Institute for Advanced Study in 1949. When Kodaira visited Weyl for the first time in his office, his hero said to him like this:

Although I may be old-fashioned...

- From Kodaira's essay: 先生曰く『私は古い(old-fashioned)かも知れないが、直交射影の方法はよくないと思う。君の論文も直交射影を用いない形に書き直した方がよい』と。これはショックであった。…ワイルの直観主義の立場では L^2 可積分な可測微分形式全体のヒルベルト空間というような大きな集合はその存在があやしい、ということであつたらしい。ワイル先生は、ゲーデルの不完全性定理によって数学の無矛盾性を証明する希望が失われた以上は、数学をあまり一般化すると矛盾に撞着するのではないかと本気で心配しておられたようである。…それにしてもヒルベルト空間がすでに危険であるという感覚は私には理解できなかった。

Although I may be old-fashioned... translated

- From Kodaira's essay: Prof. Weyl said , "I may be old-fashioned, but I think the orthogonal projection method is not good. It would be better to rewrite your paper without orthogonal projection. " This shocked me. ... It seems that the existence of the large set such as the Hilbert space of the whole of the measurable differential forms, which are L^2 measurable, is doubtful from the standpoint of Weyl 's intuitionism. I guess that Prof. Weyl was seriously worried that mathematics may confront contradictions, if it is too much generalized, as the hope of proving the consistency of mathematics had gone by Gödel's incompleteness theorem. ... At any rate I could not understand the feeling that Hilbert space was already dangerous.

The feeling of secularized mathematicians

- The following two highlighted parts represent the feeling of secularized mathematicians after the second world war.
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Not only Kodaira. Bourbaki said:

- Certainly, the vigorous attacks from the intuitionist camp have from time to time obliged not only the avant-garde mathematical schools, but even the partisans of traditional mathematics, to be on the defensive. **A well-known mathematician has admitted to being impressed by these attacks to the point that he voluntarily restricted his work to those branches of mathematics considered to be “certain”. But such cases must have been rather uncommon.** The intuitionist school, whose memory will undoubtedly survive only as a historical curiosity, has at least rendered the service of having obliged its opponents, that is to say **the vast majority of mathematicians, to clarify their own positions and to become more consciously aware of the reasons (whether logical or sentimental) for their confidence in mathematics.**

(2004 English translation “Theory of Sets” of Bourbaki’s Théorie des ensembles, 1970, p.336)

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Around 1960, secularization completed

- Around 1960, most of pre-secularized mathematicians as Weyl passed away.
- After that, no great mathematicians at the rank of Weyl cared about philosophical issues on mathematics.
- They were like Kodaira and Bourbaki.
- It is interesting that secularization of physics took place almost the same time.

Secularization of Physics 1/2

- Physics experienced a similar transformation. A notable historian of science [Helge Kragh](#) claimed in his book “Quantum Generations: A History of Physics in the Twentieth Century”, 1999, p.441, (a very good Japanese translation available 「20世紀物理学史（上下巻）」 by Okamoto, Ariga, Inaba et al.) as follows:
- One result of the postwar generational shift and that general turn toward instrumentalist and pragmatic modes of thought was that **philosophy lost its place in physics**. Many members of the earlier generation of physicists had a deep interest in philosophical questions and were sometimes inspired by philosophers in their innovative work, or they discussed competently the philosophical implications of the new physics. (continued)

Secularization of Physics 2/2

- (continued) For Plank, Bohr, Schrödinger, **Weyl**, Heisenberg, Einstein, Eddington, and many of their colleagues, philosophy was an important aspect of physics. Few of the leaders of postwar physics cared about philosophy or had more than a superficial knowledge of the field. **In the 1960's, with the death of Bohr and Schrödinger, the once proud and vital tradition of physicist-philosophers came to an end.**

Modernized Physicists and Mathematicians

- In short, Kragh maintained that the attitude of the generations of physicists after the second world war was **“No Philosophy, Please, We’re Physicists”**.
- Any physicist, who takes this attitude to philosophy, is a “modernized physicist” in my sense, just as Kodaira and Bourbaki are “modernized mathematicians”.

Difference between Physics and Math.

- Modernization was practically achieved almost at the same time in physics and mathematics.
- However, the processes were very different.
- In the case of mathematics, there was a stage of modernization at which a considerable number of great mathematicians believed that philosophical problems on mathematics would be fixed by mathematics itself.
- They believed that secular mathematics would solve the most non-secular (philosophical) problems of mathematics.
- There was not such a stage in the history of physics.
- What is the stage then?

The Stage: the Period of Hilbert's Program 1/2

- The stage is Hilbert's Program. Hilbert tried to solve many foundational problems on mathematics such as consistency and completeness by his proof theory.
- It is widely believed that this program was initiated by the discovery of antinomies of set theory.
- It's only half true. Solving the problem of set-theoretic antinomies by consistency proof is definitely an important motivation.
- However, the real origin of Hilbert's program was a plan **to prove the solvability of mathematics mathematically.**
- The solvability of mathematics means that there is a method by which all mathematical problems can be decided true or false in finite steps. In the modern words, decision method of mathematics.

The Stage: the Period of Hilbert's Program 2/2

- In his 1905 lecture, after proving his version of the completeness of propositional logic, Hilbert claimed that it is the simplest case of his grand plan to prove that there is no [Ignorabimus](#), “limitation of human wisdom”, in mathematics. He further mentioned that this plan was the starting point of all of his research on the foundations of mathematics.
- The starting point took place around 1888-9. Then he wrote about the plan of the solvability proof and other related problems, relating them to Kant's philosophy in the first of his three mathematical notebooks.
- I do not have time to explain them in detail, but there is a Web page on the findings of Hilbert's notes: [David Hilbert's Mathematical Notebooks](#), at which we will take a glance, and we have written [a paper on it](#) long time ago, but not yet published.

Solving philosophical problems by math. 1/2

- Hilbert called Brouwer and Weyl “philosophers disguised as mathematicians” in his 1931 lecture in Hamburg.
- Apparently, he was disgusted with them and philosophy. Did Hilbert change his mind from his youthful days when he mentioned Kant?
- No, he didn't!
- In 19th century, some scientists tried to draw philosophical conclusions by help of science. The greatest example is the program of Hermann von Helmholtz to rescue Kant's philosophy by physiology (pp.196-205, “The Genesis of Neo-Kantianism 1796-1880”, 2014, by F.C. Beiser).

Solving philosophical problems by math. 2/2

- The young practically unknown mathematician in 1880's was doing similar thing in mathematics, and three decades later, then he was one of the greatest mathematicians in the world, was pursuing the dream of his youth.
- It was Hilbert's Program.
- However, as Kodaira mentioned in his essay, the dream was shattered by Gödel's second incompleteness theorem.
- The best strategy left for mathematicians was just to forget about philosophy and to strengthen their confidence in mathematics.
- Anyway, there are formal systems of sets (ZF, NBG) seemingly consistent and very enough to describe their concepts and proofs.
- Then, the step of modernization was forwarded.

Gödel and Modernization

- In a sense, Gödel had the process of the modernization make a step forward.
- However, he had a different view on it.
- I claimed that after 1960's no great mathematicians at the rank of Weyl cared about philosophical issues.
- Gödel was an exception.
- He viewed the step of modernization brought by his great theorem as a step backward!

Gödel's historical view 1/3

- In Kurt Gödel Collected Works, Volume III, Unpublished Essays and Lectures, there is an interesting and enigmatic article titled “The modern development of the foundations of mathematics in the light of philosophy” (1961/?).
- Gödel presented a classification of philosophical world-views (Weltanschauungen) according to the degree and manner of how they are away from metaphysics or religion.
- Some world-views from the left side to the right side:
Left Right
skepticism theology
materialism idealism
positivism spiritualism
more metaphysical or religious →

Gödel's historical view 2/3

- Then Gödel wrote (verbatim quotations): Now it is a familiar fact, even a platitude, that the development of philosophy since the Renaissance has by and large gone from right to left-... . Particularly in physics, this development has reached a peak in our own time, ... , the possibility of knowledge of the objectivizable states of affairs is denied, and it is asserted that we must be content to predict results of observations. ... (although this predicting can be completely sufficient for practical purposes such as making television sets or atom bombs).

Gödel's historical view 3/3

- We may guess that Gödel wrote about the same historical process which I quoted from Kragh's book. Good!
- However, then, he continued as follows: **Mathematics is an a priori science, and so, it went in the opposite direction!!**
- He even claimed that Hilbert's program was a strange hybrid (merkwürdige Zwitterding) attempt to rescue the rightward aspects of mathematics by the leftward proof theory.
- The Hilbertian combination of materialism and aspects of classical mathematics thus proves to be impossible. (verbatim)
- It was as if he was saying the failure of Hilbert's program by his incompleteness theorems was a destiny.

The Origin of My Research

- I happened to read the above article 19 years ago.
- Then I was an amateur historian, who was a professor of applied mathematics and software engineering.
- Just by chance, I was about to study Weber sociology for understanding of my problems in software engineering.
- Then, it became my fascination to understand the process of history that Gödel described by Weber's modernization theory, and it made me even a professional historian.
- It took nearly 19 years to reach a satisfactory view.
- It is my idea of Modernization of Mathematics.

Realities for Mathematicians 1/3

- I claimed, after 1960's, no great mathematicians at the rank of Weyl cared about philosophical issues on mathematics.
- However, there was an exception at least: Gödel!
- He was known as a philosopher of mathematics, as he had claimed "Gödel's Platonism". He believed that transfinite sets exist just as in the same sense as physical bodies exist.
- It is doubtful that Bourbaki members thought in the same way.
- For most mathematicians, axiomatic set theory is no more than a lingua franca (common language) to describe their own "realities".

Realities for Mathematicians 2/3

- They have their own realities, subjects of research, in their minds, e.g. complex manifolds, Hilbert spaces, spectrums of rings, etc.
- They do not care about “materials” of their realities, as modern mathematicians care about only structures, “formal causes” in a sense.
- Thus, some of them prefer category theory to set theory, as they can do mathematics without talking about materials of their subjects of research.

Realities for Mathematicians 3/3

- We may consider axiomatic set theories or the like, e.g. NBG, ZF, Tarski-Grothendieck set theory, elementary topos, etc., as examples of the formal rationality in mathematics.
- Thinking so, mathematicians' realities become “re-enchantment through disenchantment”, a sociological issue developed after Weber sociology.
- Anyway, it is impossible to do research of mathematics without thinking that subjects of research really exist so that you can even touch them. As an ex-mathematician, I deeply believe in this from my own experiences.
- However, as a historical sociologist or historian, I see this as a re-enchantment through disenchantment.

Modernization of Mathematics 1/3

- From this standpoint, the whole development of the foundations of mathematics, modernization of mathematics, looks like the following:
- Since Riemann, there was a metaphysical trend in mathematics.
- It was questioned by 19th century rationalists such as Kummer, Kronecker and Weierstrass, but the history proved they were wrong and Riemann was right.
- Metaphysical ideas such as Riemannian surface and infinite set made mathematical researches much more **productive**.
- It was contradictory, as Gödel illustrated in the 1961 article that I quoted.
- Thus, as Gödel pointed out, it was natural that contradictions were found in the metaphysical foundation, i.e. set theory.

Modernization of Mathematics 2/3

- What early 20th century rationalists did was to control too strong a power of the metaphysics in mathematics by the formal rationality in mathematics, i.e. formal systems.
- Although Hilbert's program, which was an attempt to validate this disenchanting metaphysics by the left most standpoint, failed, mathematicians did not mind using the disenchanting metaphysics.
- Reason: they have their own metaphysical realities in their minds and the disenchanting metaphysics is no more than a lingua franca for mathematics for most of mathematicians.

Modernization of Mathematics 3/3

- Spanish historian José Ferreirós pointed out that the early set theories were considered as improved Aristotelian logic.
 - Dedekind called his version of set theory in 1888 booklet “Was sind und was sollen die Zahlen?” Logik. Thus he identified the singleton of an entity with the entity itself in the booklet.
- Namely, axiomatic set theory is a rationalized metaphysics, Aristotelian logic improved by the algebraic and formal methods initiated by George Boole.
- It is a land which was once a part of the kingdom of philosophy but is now a part of the republic of mathematics.
- Since they can find enough metaphysics for their research in their own land, contemporary mathematicians ignore philosophy.
- The modernization of mathematics was achieved in this way.