

Church without dogma: axioms for computability

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Church's and Turing's theses dogmatically assert that an informal notion of computability is captured by a particular mathematical concept. I present an analysis of computability that leads to precise concepts, but dispenses with theses.

To investigate computability is to analyze processes that can in principle be carried out by calculators. Drawing on this lesson we owe to Turing and recasting work of Gandy, I formulate finiteness and locality conditions for two types of calculators, human computing agents and mechanical computing devices; the distinctive feature of the latter is that they can operate in parallel.

The analysis leads to axioms for discrete dynamical systems (representing human and machine computations) and allows the reduction of models of these axioms to Turing machines. Cellular automata and a variety of artificial neural nets can be shown to satisfy the axioms for machine computations.

References:

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