Dear Sir.

I have to thank you very much for the last papers: the proof for the finite number of the invariants of a binary quantic is very much simplified (& indeed becomes beautifully simple & easy) if instead of invariants you consider seminvariants; in fact writing $(a, b, .. \between x, y)^n$ $= a(x-\alpha y)(x-\beta y)\ldots$, the general form of an irrational seminvariant is $S = \{a(\alpha-\beta)\}^p\{a(\alpha-\gamma)\}^q\{a(\beta-\gamma)\}^r \ldots$, and we obtain all the rational seminvariants by considering the like symmetrical forms, or simply prefixing a Σ to the foregoing expression. But by the equation of differences $\{a(\alpha-\beta)\}^{n(n-1)} + X_1\{a(\alpha-\beta)\}^{n(n-1)-2} + \&c = 0$ where the coefficients X_1, X_2 &c are functions of seminvariants: hence in the expression for S it is only necessary to attend to forms wherein each of the exponents $p, q, \ldots$ is at most $= n(n-1)$, and the number of such forms is finite. Thus every seminvariant is of the form $A\Omega_1 + B\Omega_2 + \ldots$ where the number of forms $A, B, \ldots$ is finite, and where $\Omega_1, \Omega_2 \ldots$ denote rational & integral functions of the coeff^ts $X_1, X_2 ..$ of the equation of differences, or what is the same thing of a finite number of seminvariants. I propose to write this out for the Annalen. I congratulate you very heartily on this your solution of what has been so long a difficulty.

Believe me dear Sir Yours very sincerely

A. Cayley

Cambridge 22^nd Jan^y. 1889.

Dear Sir,

I am much obliged for your two letters: My difficulty was an à priori one, I thought that the like process should be applicable to seminvariants, which it seems it is not: & I now quite see that, for invariants your step from $i = a_1 i_1 \ldots + a_m i_m$ to $i = I_1 i_1 \ldots + I_m i_m$ is quite right. I think you have found the true solution of a great problem.

But the proof of theorem I seems to me to need revision. I understand "form" to mean <u>homogeneous</u> form: thus $n=1$, the forms φ of theorem I are mere powers of x: and the like for ψ &c.

Suppose to fix the ideas $\varphi_1, \varphi_2 \,..\, \varphi_m = x^1, x^2 \,..\, x^m$ and $\psi_1, \psi_2 \,..\, \psi_m = x^1, x^4, \ldots x^{m^2}$: I is obviously true. we have ~~x^m~~ ~~$\varphi_m = \alpha_1\varphi_1$~~ $\varphi_k = \alpha_{k1}\varphi_1$ that is ~~$x^m = x^{m-1}.\,x$~~ $x^k = x^{k-1}.\,x$; and hence $\Psi_k = x^{k^2} - x^{k-1}.\,x, \; = x^{k^2} - x^k$, which is not homogeneous, & hence in assuming the theorem to be true for the functions Ψ you are in effect assuming it, not for the value $n=1$, but for the value $n=2$: so that you do not in this way pass from Theorem I, $n=1$ to Theorem II, $n=1$.

I think I see my way to a proof of theorem I, starting with the general case of n variables.

I wrote out my proof for the Seminvariants (founded on yours in the Math. Annalen) & sent it to Prof. Klein: it seemed to me all right, and I trust it will appear so to you when it is published.

Believe me dear Sir, Yours very sincerely

A. Cayley

Cambridge 30th Jan. 1889.